Rob Salgado (PHYS 1120)

LAB: Electric Potential and Field (PhET and Desmos)

- OBJECTIVE: To investigate the properties of the Electric Potential and Electric Field.

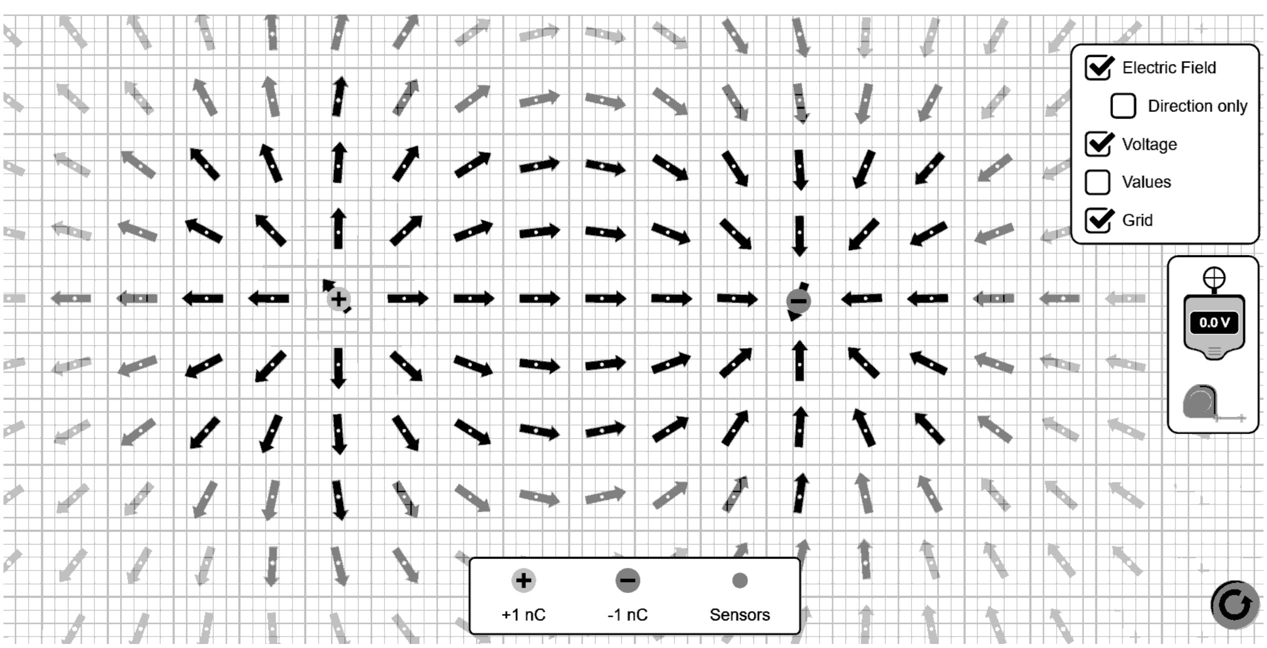
1. Visit https://phet.colorado.edu/en/simulations/charges-and-fields and
   • set up the following configuration of charges: **an electric dipole**
   (equal-magnitude, opposite-sign charges separated by a fixed displacement).
   • Turn on the checkboxes for Grid and for Values.
   The positive-charge is at $x = -3.5$ boxes (-175 cm using the tape-measure from the central vertical), and the negative charge is at $x = +3.5$ boxes.
   [Two adjacent boxes have a width of 100cm.
   At the center of each box is an arrow that is essentially a "compass for the electric field", which can be modeled by a small electric dipole and realized by, e.g., grass-seeds.]
   (You want to match this image below because you will mark it up based on your use of the PhET simulation.)

   Use the *Voltage probe* to identify the "equipotential of +4.000 V" as follows:
   start in the upper left corner and approach the positive charge,
   when you find a point whose electric potential is within $\pm 0.050\ V$ of +4.000 V,
   • mark it on the image below *[where you can use the grid of arrows to locate your points]*
   • and Drag an "[Electric Field] Sensor" to that location.
   Then find at least <u>10 points</u> more points (enough to suggest a sketch of a closed loop for +4.000 V) to be marked on the image and to have a Sensor dropped there.
   • When you have enough points on your image below to sketch the equipotential, do so.
   • Mark that loop as "+4 V" on the image below.

   • Only after you have marked your last point and sketched your equipotential below, you may now push the pencil button.

   • Repeat for "+2 V".

   • Locate <u>one</u> point where $V = +6$ V. • Then, push the pencil button. *(You don't need to mark the paper.)* •Repeat for +8V, +10V, then +2 V, 0 V, −2 V, −4 V, −6 V, −8 V, and −10 V.
   •Take a screen shot of the simulation for the lab report. Keep the PhET window open.



Keep the PhET window open and untouched (make sure you have taken your screenshot of it).
If you modify the PhET, you may lose some measurements. So, don't modify it.
Let's try to reproduce this in Desmos…

2. • Go to **https://www.desmos.com/calculator** .
Zoom in so that the vertical range is about $-2.5 < y < +2.5$.
(You should use the wrench (in the upper right corner) to "Zoom Square", which is shown when the axes are not equally-scaled.)

• In the first cell, type a double-quote character followed by your name(s).

• Next, set up some convenient constants (on separate lines)
$u_{nC} = 10^{-9}$, $k = 9 \cdot 10^{9} \cdot u_{nC}$
Desmos is case-sensitive! And Desmos is picky about subscripts and superscripts!

• Set up the charges (on separate lines):
$Q_1 = 1$, $P_1 = (-1.75,0)$ and $Q_2 = -1$, $P_2 = (1.75,0)$.
(P_1 and P_2 are draggable points [indicated by the cross of arrows], and Q_1 and Q_2 are the charges in nC (thanks to u_{nC}) and are tunable by sliders.
But let's keep the values as shown here.)
You can change colors by long-pressing the circle for each entry.

• Set the label on P_1 to read ${Q_1} and the label P_2 to read ${Q_2} .
If the resulting label is displayed as a **?** *then you made an error somewhere. Check upper-case vs lower-case, and check spelling.*

• Let's use Z for our "probe" location (which will be draggable) and initialize it as:
$Z = (-0.5,1.0)$

• Let's write a function for the **"Electric Potential due to the two charges"** (in Volts) for any point (x, y) in the plane:

$$V_1(x,y) = \frac{kQ_1}{distance\big((x,y),P_1\big)}$$
$$V_2(x,y) = \frac{kQ_2}{distance\big((x,y),P_2\big)}$$
$$V(x,y) = V_1(x,y) + V_2(x,y)$$

[Note that the Desmos function $distance\big((x,y),P_1\big)$ is equivalent to $\sqrt{(x - P_1.x)^2 + (y - P_1.y)^2}$ (using **sqrt** and **.x** and **.y** for the coordinates of P_1).]

• Let's define V_Z to be the electric potential value measured by our "probe":
$V_Z = V(\ Z.x\ , Z.y\ )$
where we have supplied the x- and y-coordinates of Z as arguments to $V(x, y)$.

• Let's use this value of V_Z as the label to Z. Enter ${V_Z} for Z's label.
If the resulting label is displayed as a **?** *then you made an error somewhere.*
Try it out. Drag Z around to locate points where $V = +4\ V$.

• Let's get Desmos to find all of those points where $V = +6\ V$. Write in a new cell:
$6 = V(x,y)$
which is an implicit equation for (x, y). It has to be written this way, or else it will be interpreted by Desmos as redefinition of our V-function.

• Next, in a new cell, write: $4 = V(x,y)$ …. in another cell, $2 = V(x,y)$

- Rather use do this for each number, we can use a Desmos list to do many of them in one cell:
$[2, 4, \ldots 10] = V(x, y)$
which counts from 2, in steps of (4-2)=2, up to 10.

- To get the zero-equipotential and the negative-equipotentials, include these two lines
$0 = V(x, y)$
$[-2, -4, \ldots -10] = V(x, y)$
Make the zero-equipotential a dotted-line *by long-pressing the circle.*
Make the negative-equipotentials dashed-lines.

- Click the "Share this graph button" in the upper right corner.
Copy that URL for your report.

3. It might be good to see how the equipotentials are affected when new charge is introduced nearby. Set the value for Q_2 to be zero.
In a new cell, type in
$[2,4, \ldots 10] = V_1(x, y)$
which shows the equipotentials for the left-charge alone.
Then, slowly vary the Q_2 slider from 0 nC to 4 nC, then backwards to -4 nC, then back to -1 nC. *(You can automate this by changing the limits of the Q_2-slider to " -4 to $+4$", clicking the button-with-arrows on the left slide of the slider and setting the animation speed to 0.1x , then press the Play button in the circle above.)*
Think about the elevation graph and contour graph representations of electric potential.

- Return the charges to the original values and locations.
Disable the circle for the cell with $[2,4, \ldots 10] = V_1(x, y)$.

ELECTRIC FIELDS

4. Let's try to include the **Electric Field vector** in our visualization.
At the bottom of your working Desmos code, copy-paste this URL into a new cell:
`https://www.desmos.com/calculator/3zosseby8r`
It makes a folder with "vector-drawing functions" that were inspired by another Desmos user. (You don't need to open it. But you can look inside if you wish.)
- Here's the Electric Field at location (x, y) due to charge Q_1 at location P_1

$$E_1(x, y) = \frac{kQ_1}{distance\big((x, y), P_1\big)^2} \cdot \frac{\big((x, y) - P_1\big)}{distance\big((x, y), P_1\big)}$$

The first factor is the radial component (centered at the charge's location P_1).
The second factor is the radial unit-vector pointing away from the source $\hat{r}_{\text{away from } P_1}$.
- Write an analogous function for $E_2(x, y)$,
then write the vector sum $E_{vec}(x, y) = E_1(x, y) + E_2(x, y)$.

- To see the vector, first introduce a tunable scaling factor [for visualization purposes]
$f = -3$
then enter in a new cell
$G_{vecAt}\big(Z,\ E_{vec}(Z.x, Z.y) \cdot 2^f\ , 0.1\big)$

- Follow an equipotential with the probe Z. **Comment on the relationship between**
  - the *direction* of the electric field vector and the *tangent line* to the equipotential
  DESCRIBE: ____
  - the *direction* of the electric field vector points toward
  CHOOSE: ?increasing or ?decreasing electric potential.
  - the *magnitude* of the electric field vector is
  CHOOSE: ?large or ?small where the *spacing of the equal-increment potentials* is small.

• To see the electric field vector as the VECTOR-SUM of the two individual electric fields, enter in new cells:
$G_{vecAt}\left(Z,\ E_1(Z.x, Z.y)\cdot 2^f\ ,0.1\right)$
$G_{vecAt}\left(Z,\ E_2(Z.x, Z.y)\cdot 2^f\ ,0.1\right)$
Change the colors of these electric field vectors to match the color of the corresponding source-charges. *You can change the colors and thickness of the vectors by long-pressing the circle.*

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SOME MULTIVARIABLE CALCULUS

5. Although we have explicit formulas for the electric field and electric potential for point charges, it turns out that they are related by **[multivariable] calculus.**
Given the electric potential function,
the **"electric field vector"** is equal to the **"minus the gradient"** of the **"electric potential".**
Symbolically, this is written as $\vec{E}(x,y) = -\vec{\nabla}\, V(x,y)$, which is a compact notation for

$$E_x(x,y) = -\frac{d}{dx}V(x,y)$$
$$E_y(x,y) = -\frac{d}{dy}V(x,y)$$

•In Desmos, enter these in new cells

$$\boldsymbol{E_{\text{asGrad}}(x,y) = \left(-\frac{d}{dx}V(x,y), -\frac{d}{dy}V(x,y)\right)}$$
$$G_{vecAt}\left(Z,\ E_{\textbf{asGrad}}(\,Z.x\,, Z.y\,)\cdot 2^f\ ,0.1\right)$$

The last line draws a vector using the $E_{\boldsymbol{asGrad}}$ you just defined.
Since this agrees with the explicit formula E_{vec}, it's hard to see.
•You can change the linestyle to dotted or dashed and the colors *by long-pressing the circle.*

6. One of the requirements for the *mathematical existence of the electric potential function* is that the **work-done by the electrostatic force is independent of the path**
("the electrostatic force is said to be ***conservative***").
Given the electric field,
the **"electric potential"** at a location in space is equal to the **"line-integral of the electric field from infinity to that location in space, independent of the path".**
Symbolically, this is written as $V(x,y) = -\int_{\infty}^{(x,y)} \vec{E}\cdot d\vec{l}$.
•In Desmos, CAREFULLY (with capitals and lower-case and .x and .y)
enter these in new cells
$\boldsymbol{V_{asInt1}(X,Y) = -\left(\int_{\infty}^{1}(\ E_{vec}(sX,sY).x\ \ X\ +\ E_{vec}(sX,sY).y\ \ Y\ )\,ds\right)}$
$\boldsymbol{V_{asInt2}(X,Y) = -\left(\int_{-\infty}^{0} E_{vec}(0,y)\,.y\ \ dy\ + \int_{0}^{1}(\ E_{vec}(sX,sY).x\ X + E_{vec}(sX,sY).y\ Y\ )\,ds\right)}$
then enter these in new cells
$V_{asInt1}(Z.x, Z.y)$
$V_{asInt2}(Z.x, Z.y)$
These should agree with V_Z defined earlier using V(x,y)… *although they are slow to compute.*
(Path 1 is along the line joining position Z and the origin O, starting from a large position vector $s(\overrightarrow{OZ})$ to position Z. Path 2 is along the negative-y-axis from infinity to the origin, then to position Z.) Of course, this is only a demonstration—not a proof—of the independence of path.

7. Click the "Share this graph button" in the upper right corner.
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